## SECTION-I

*General Instructions:*

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.

**Q. 1.** Consider a triangle $\Delta$ whose two sides lie on the X-axis and the line $x + y + 1 = 0$. If the orthocenter of $\Delta$ is (1, 1), then the equation of the circle passing through the vertices of the triangle $\Delta$ is

**(A)** $x^2 + y^2 - 3x + y = 0$

**(B)** $x^2 + y^2 + x + 3y = 0$

**(C)** $x^2 + y^2 + 2y - 1 = 0$

**(D)** $x^2 + y^2 + x + y = 0$

**Q. 2.** The area of the region

$$\left\{(x, y) : 0 \leq x \leq \frac{9}{4}, 0 \leq y \leq 1, x \leq 3y, x + y \geq 3\right\}$$

is

**(A)** $\frac{11}{32}$ **(B)** $\frac{35}{96}$

**(C)** $\frac{37}{96}$ **(D)** $\frac{13}{32}$

**Q. 3.** Consider three sets $E_1 = \{1, 2, 3\}, F_1 = \{1, 3, 4\}$ and $G_1 = \{2, 3, 4, 5\}$. Two elements are chosen at random, without replacement, from the set $E_1$, and let $S_1$ denote the set of these chosen elements. Let $E_2 = E_1 - S_1$ and $F_2 = F_1 \cup S_1$. Now two elements are chosen at random, without replacement, from the set $F_2$ and let $S_2$ denote the set of these chosen elements.

Let $G_2 = G_1 \cup S_2$. Finally, two elements are chosen at random, without replacement, from the set $G_2$ and let $S_3$ denote the set of these chosen elements.

Let $E_3 = E_2 \cup S_3$. Given that $E_1 = E_3$, let p be the conditional probability of the event $S_1 = \{1, 2\}$. Then the value of p is

**(A)** $\frac{1}{5}$ **(B)** $\frac{3}{5}$

**(C)** $\frac{1}{2}$ **(D)** $\frac{2}{5}$

**Q. 4.** Let $\theta_1, \theta_2, \ldots, \theta_{10}$ be positive valued angles (in radian) such that

$\theta_1 + \theta_2 + \ldots + \theta_{10} = 2\pi$. Define the complex numbers $z_1 = e^{i\theta_1}$, $z_k = z_{k-1}e^{i\theta_k}$ for $k = 2, 3, \ldots, 10$, where $i = \sqrt{-1}$. Consider the statements P and Q given below:

$$P : |z_2 - z_1| + |z_3 - z_2| + \ldots + |z_{10} - z_9| + |z_1 - z_{10}| \leq 2\pi$$

$$Q : |z_2^2 - z_1^2| + |z_3^2 - z_2^2| + \ldots + |z_{10}^2 - z_9^2| + |z_1^2 - z_{10}^2| \leq 4\pi$$

Then,

**(A)** P is TRUE and Q is FALSE

**(B)** Q is TRUE and P is FALSE

**(C)** both P and Q are TRUE

**(D)** both P and Q are FALSE

**Question Stem for Question Nos. 5 and 6**

**Question Stem**

Three numbers are chosen at random, one after another with replacement, from the set $S = \{1, 2, 3, \ldots, 100\}$. Let $p_1$ be the probability that the maximum of chosen numbers is at least 81 and $p_2$ be the probability that the minimum of chosen numbers is at most 40.

**Q. 5.** The value of $\frac{625}{4} p_1$ is ______.

**Q. 6.** The value of $\frac{125}{4} p_2$ is ______.

**Question Stem for Question Nos. 7 and 8**

**Question Stem**

Let $\alpha$, $\beta$ and $\gamma$ be real numbers such that the system of linear equations

$$x + 2y + 3z = \alpha$$
$$4x + 5y + 6z = \beta$$
$$7x + 8y + 9z = \gamma - 1$$

is consistent. Let $|M|$ represent the determinant of the matrix

$$M = \begin{bmatrix} \alpha & 2 & \gamma \\ \beta & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

Let P be the plane containing all those $(\alpha, \beta, \gamma)$ for which the above system of linear equations is consistent, and D be the square of the distance of the point (0, 1, 0) from the plane P.

**Q. 7.** The value of $|M|$ is ______ .

**Q. 8.** The value of D is _______ .

**Question Stem for Question Nos. 9 and 10**

**Question Stem**

Consider the lines $L_1$ and $L_2$ defined by

$L_1: x\sqrt{2} + y - 1 = 0$ and $L_2: x\sqrt{2} - y + 1 = 0$

For a fixed constant $\lambda$, let C be the locus of a point P such that the product of the distance of P from $L_1$ and the distance of P from $L_2$ is $\lambda^2$. The line $y = 2x + 1$ meets C at two points R and S, where the distance between R and S is $\sqrt{270}$.

Let the perpendicular bisector of RS meet C at two distinct points R' and S'. Let D be the square of the distance between R' and S'.

**Q. 9.** The value of $\lambda^2$ is _______.

**Q. 10.** The value of D is _______.

## SECTION-III

*General Instructions:*

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).

**Q. 11.** For any $3 \times 3$ matrix M, let $|M|$ denote the determinant of M. Let

$$E = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 8 & 13 & 18 \end{bmatrix}, P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \text{ and}$$

$$F = \begin{bmatrix} 1 & 3 & 2 \\ 8 & 18 & 13 \\ 2 & 4 & 3 \end{bmatrix}$$

If Q is a nonsingular matrix of order $3 \times 3$, then which of the following statements is (are) TRUE ?

**(A)** $F = PEP$ and $P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

**(B)** $|EQ + PFQ^{-1}| = |EQ| + |PFQ^{-1}|$

**(C)** $|(EF)^3| > |EF|^2$

**(D)** Sum of the diagonal entries of $P^{-1}EP + F$ is equal to the sum of diagonal entries of $E + P^{-1}FP$

**Q. 12.** Let $f : R \rightarrow R$ be defined by

$$f(x) = \frac{x^2 - 3x - 6}{x^2 + 2x + 4}$$

Then which of the following statements is (are) TRUE ?

**(A)** $f$ is decreasing in the interval $(-2, -1)$

**(B)** $f$ is increasing in the interval $(1, 2)$

**(C)** $f$ is onto

**(D)** Range of $f$ is $\left[-\frac{3}{2}, 2\right]$

**Q. 13.** Let E, F and G be three events having probabilities

$P(E) = \frac{1}{8}$, $P(F) = \frac{1}{6}$ and $P(G) = \frac{1}{4}$, and let

$P(E \cap F \cap G) = \frac{1}{10}$.

For any event H, if $H^c$ denotes its complement, then which of the following statements is (are) TRUE ?

**(A)** $P(E \cap F \cap G^c) \leq \frac{1}{40}$

**(B)** $P(E^c \cap F \cap G) \leq \frac{1}{15}$

**(C)** $P(E \cup F \cup G) \leq \frac{13}{24}$

**(D)** $P(E^c \cap F^c \cap G^c) \leq \frac{5}{12}$

**Q. 14.** For any $3 \times 3$ matrix M, let $|M|$ denote the determinant of M. Let I be the $3 \times 3$ identity matrix. Let E and F be two $3 \times 3$ matrices such that $(I - EF)$ is invertible. If $G = (I - EF)^{-1}$, then which of the following statements is (are) TRUE ?

**(A)** $|FE| = |I - FE||FGE|$

**(B)** $(I - FE)(I + FGE) = I$

**(C)** $EFG = GEF$

**(D)** $(I - FE)(I - FGE) = I$

**Q. 15.** For any positive integer $n$, let $S_n : (0, \infty) \to R$ be defined by

$$S_n(x) = \sum_{k=1}^{n} \cot^{-1}\left(\frac{1+k(k+1)x^2}{x}\right),$$

where for any $x \in R$, $\cot^{-1}(x) \in (0, \pi)$ and $\tan^{-1}(x) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ Then which of the following statements is (are) TRUE ?

**(A)** $S_{10}(x) = \frac{\pi}{2} - \tan^{-1}\left(\frac{1+11x^2}{10x}\right)$ for all $x > 0$

**(B)** $\lim_{x\to\infty} \cot(S_n(x)) = x$, for all $x > 0$

**(C)** The equation $S_3(x) = \frac{\pi}{4}$ has a root in $(0, \infty)$

**(D)** $\tan(S_n(x)) \leq \frac{1}{2}$, for all $n \geq 1$ and $x > 0$

**Q. 16.** For any complex number $\omega = c + id$, let $\arg(\omega) \in (-\pi, \pi]$, where $i = \sqrt{-1}$. Let $\alpha$ and $\beta$ be real numbers such that for all complex numbers $z = x + iy$ satisfying $\arg\left(\frac{z+\alpha}{z+\beta}\right) = \frac{\pi}{4}$, the ordered pair $(x, y)$ lies on the circle

$$x^2 + y^2 + 5x - 3y + 4 = 0$$

Then which of the following statements is (are) TRUE ?

**(A)** $\alpha = -1$ **(B)** $\alpha\beta = 4$

**(C)** $\alpha\beta = -4$ **(D)** $\beta = 4$

## SECTION-IV

*General Instructions:*

- This section contains **THREE (03)** questions.
- The answer to each question is a **NON–NEGATIVE INTEGER.**

**Q. 17.** For $x \in R$, the number of real roots of the equation

$3x^2 - 4|x^2 - 1| + x - 1 = 0$ is ____ .

**Q. 18.** In a triangle ABC, let $AB = \sqrt{23}$, $BC = 3$ and $CA = 4$. Then the value of $\frac{\cot A + \cot C}{\cot B}$ is

**Q. 19.** Let $\vec{u}$, $\vec{v}$ and $\vec{w}$ be vectors in three–dimensional space, where $\vec{u}$ and $\vec{v}$ are unit vectors which are not perpendicular to each other and

$\vec{u} \cdot \vec{w} = 1, \vec{v} \cdot \vec{w} = 1\ \vec{w} \cdot \vec{w} = 4$

If the volume of the parallelopiped, whose adjacent sides are represented by the vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$, is $\sqrt{2}$, then the value of $|3\vec{u} + 5\vec{v}|$ is _______ .

# Answers

| Q. No. | Answer | Topic Name | Chapter Name |
|---|---|---|---|
| 1 | (B) | Special Points in Triangles | Conic Section |
| 2 | (A) | Straight Line and a Point | Conic Section |
| 3 | (A) | Conditional Probability | Probability |
| 4 | (C) | Geometry of Complex Numbers | Complex Number |
| 5 | [76.35] | Algebra of Probabilities | Probability |
| 6 | [24.50] | Algebra of Probabilities | Probability |
| 7 | [1] | Systems of Linear Equations | Matrices and Determinants |
| 8 | [1.5] | Plane and a Point | Three Dimensional Geometry |
| 9 | [9] | Distance of a Point from a Line | Point and Straight Line |
| 10 | [77.14] | Interaction between two Lines | Point and Straight Line |
| 11 | (A, B, D) | Inverse of a Matrix | Matrices and Determinants |
| 12 | (A, B) | Maxima and Minima | Application of Derivatives |
| 13 | (A, B, C) | Algebra of Probabilities | Probability |
| 14 | (A, B, C) | Properties of Matrix Operations | Matrices and Determinants |
| 15 | (A, B) | Properties of Inverse Trigonometric Functions | Inverse Trigonometric Functions |
| 16 | (B, D) | Geometry of Complex Numbers | Complex Number |
| 17 | [4] | Types of Functions | Functions |
| 18 | [2] | Relations between Sides and Angles of a Triangle | Properties of Triangle |
| 19 | [7] | Triple Products | Vector Algebra |

# ANSWERS WITH EXPLANATION

**1. Option (B) is correct.**

Given: A triangle $\Delta$ with one side on X-axis and other on $x + y + 1 = 0$

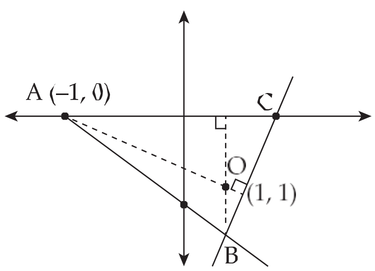
Orthocenter of $\Delta$ is (1, 1)

Let the triangle be ABC

Point A is the intersection point of X-axis and $x + y + 1 = 0$

$\Rightarrow$ coordinates of A $\equiv (-1, 0)$

Let orthocenter be O,




Let B $\equiv (x_2, y_2) \rightarrow c = (x_2, y_2)$

Since, the orthocenter is the intersection point of the altitudes.

$\Rightarrow$ BO $\perp$ AC and AO $\perp$ BC

Slope of AC = slope of X-axis = 0

As we know, if slope of two perpendicular lines be $m_1$ and $m_2$, then $m_1 . m_2 = -1$

$\therefore$ BO $\perp$ AC $\Rightarrow$ $\quad m_{BO} . m_{AC} = -1$

$\Rightarrow m_{BO} = \infty \qquad \{\because m_{AC} = 0\}$

Also, slope of a line passing through $(\alpha_1, \beta_1)$ and $(\alpha_2, \beta_2)$ is

$\Rightarrow \quad x_1 - 1 = 0$

$$m = \frac{\beta_2 - \beta_1}{\alpha_2 - \alpha_1}$$

$$\Rightarrow \quad m_{BO} = \frac{y_1 - 1}{x_1 - 1} = \infty$$

$\Rightarrow \quad x_1 = 1$

$\therefore$ B passes through $x + y + 1 = 0$

$\Rightarrow \quad y_1 = -1 - x_1$

$\Rightarrow \quad y_1 = -2$

So, coordinates of B are (1, –2)

Similarly, AO $\perp$ BC

$\Rightarrow \quad m_{AO} . m_{Bc} = -1$

$$m_{BC} = \frac{-2 - y_2}{1 - x_2}$$

and $$m_{AO} = \frac{(1-0)}{(1-(-1))} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{1}{2}\left(\frac{y_2 + 2}{x_2 - 1}\right) = -1$$

$\Rightarrow \quad y_2 + 2 = -2x_2 + 2$

$\Rightarrow \quad y_2 = -2x_2$

$\therefore$ C passes through X-axis

$\Rightarrow \quad y_2 = 0$

$\therefore \quad x_2 = 0$ and $y_2 = 0$

So, coordinates of C are (0, 0)

Since, the centroid of a triangle ABC with vertices $(a_1, b_1)$, $(a_2, b_2)$ is given by

$$\left(\frac{a_1 + a_2 + a_3}{3}, \frac{b_1 + b_2 + b_3}{3}\right)$$

$$\therefore \quad \text{Centroid of } \Delta ABC = \left(\frac{-1+1+0}{3}, \frac{0-2+0}{3}\right) = \left(0, \frac{-2}{3}\right)$$

Let centroid be M

As we know, M $\left(0, \frac{-2}{3}\right)$ divides the line segment joining circumcenter N($h$, $k$) and orthocenter O(1, 1) in the ratio 1 : 2.

$$N(h, k)\overset{1:2}{\underset{M\left(0, \frac{-2}{3}\right)}{\text{———}}}O(1, 1)$$

As we know, by section formula the coordinates of point dividing a line passing through $(x_1, y_1)$ and $(x_2, y_2)$ in the ratio $m : n$, is given by.

$$\left(\frac{mx_2+nx}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

$$\Rightarrow \left(0, \frac{-2}{3}\right) = \left(\frac{2h+1}{1+2}, \frac{2k+1}{1+2}\right)$$

$$\Rightarrow \left(0, \frac{-2}{3}\right) = \left(\frac{2h+1}{3}, \frac{2k+1}{3}\right)$$

$\Rightarrow$ $2h + 1 = 0$ and $2k + 1 = -2$

$\Rightarrow$ $h = \frac{-1}{2}$ and $k = \frac{-3}{2}$

So, circumcenter N $(h, k) \equiv \left(\frac{-1}{2}, \frac{-3}{2}\right)$

$\Rightarrow$ center of the required circle is N $\left(\frac{-1}{2}, \frac{-3}{2}\right)$ and radius is CN

$$\Rightarrow \text{radius CN} = \sqrt{\left(\frac{-1}{2}-0\right)^2+\left(\frac{-3}{2}-0\right)^2}$$

$$= \sqrt{\frac{1}{4}+\frac{9}{4}}$$

$$= \sqrt{\frac{10}{2}} \text{ units}$$

Equation of circle with center $(a, b)$ and radius $r$ of is

$(x - a)^2 + (y - b)^2 = r^2$

So, required equation of the circle

$$= \left(x-\left(\frac{-1}{2}\right)\right)^2+\left(y-\left(\frac{-3}{2}\right)\right)^2 = \left(\frac{\sqrt{10}}{2}\right)^2$$

$$= \left(x+\frac{1}{2}\right)^2+\left(y+\frac{3}{2}\right)^2 = \frac{10}{4}$$

$$\Rightarrow x^2 + x + \frac{1}{4} + y^2 + 3y + \frac{9}{4} = \frac{10}{4}$$

$$\Rightarrow x^2 + y^2 + x + 3y = 0$$

**Hints:**

(i) Find the coordinates of vertices of triangle using condition of slope for perpendicular lines.

(ii) Use the concept that a centroid divided the line segment joining circumcenter and orthocenter of a triangle in the ratio 1 : 2.

(iii) The equation of circle haring center (a, b) and radius $r$ is given by $(x - a)^2 + (y - b)^2 = r^2$

**Shortcut Method:**

Given: orthocenter of $\Delta$ is (1, 1)

As we know, the mirror image of orthocenter in side of a triangle lies on the circumcircle.

Image of (1, 1) in X-axis is (1, –1)

And image of (1, 1) in $x + y + 1 = 0$ is

$$\frac{x-1}{1} = \frac{y-1}{1} = -\frac{2(1+1+1)}{1+1}$$

$\Rightarrow$ $x - 1 = y - 1 = -3$

$\Rightarrow$ $x = -2$ and $y = -2$

$\Rightarrow$ image of (1, 1) in $x + y + 1 = 0$ is (–2, –2)

Now, the required circle will pass through (1, –1) and (–2, –2)

According to option only (B) $x^2 + y^2 + x + 3y = 0$ satisfies the condition

So, required equation is $x^2 + y^2 + x + 3y = 0$

**2. Option (A) is correct.**

Given: The region is:

$$\left\{(x, y): 0 \le x \le \frac{9}{4}, 0 \le y \le 1, x \ge 3y, x + y \ge 2\right\}$$

The intersection point of $x + y = 2$ and $x = 3y$ be J.

$\Rightarrow$ $3y + y = 2$

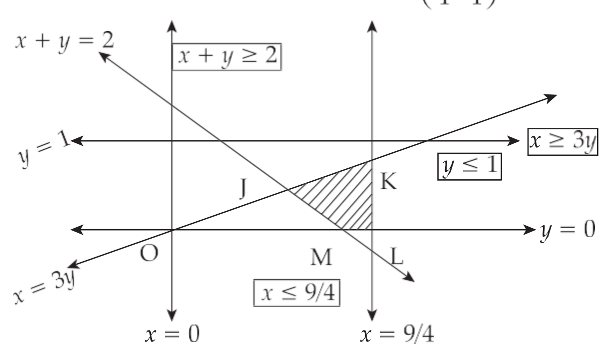
$\Rightarrow$ $y = \frac{2}{4} = \frac{1}{2}$

$\Rightarrow$ $x = \frac{3}{2}$

$\therefore$ Coordinates of point J are $\left(\frac{3}{2}, \frac{1}{2}\right)$

Similarly, intersection point of $x = \frac{9}{4}$ and $x - 3y = 0$ be K

$\Rightarrow$ Coordinates of point K are $\left(\frac{9}{4}, \frac{3}{4}\right)$



Coordinates of point M is intersection of $x + y = 2$ and $y = 0$

$\therefore$ Coordinates of M are (2, 0)

and coordinates of L is intersection of

$$y = 0 \text{ and } x = \frac{9}{4}$$

$\therefore$ coordinates of L are $\left(\frac{9}{4}, 0\right)$

So, the required region is JKLM with

$$J\left(\frac{3}{2},\frac{1}{2}\right),\ K\left(\frac{9}{4},\frac{3}{4}\right),\ L\left(\frac{9}{4},0\right),\ M(2,0)$$

Required area = area of $\Delta$JKM + area of $\Delta$MKL

$$\text{Area of } \Delta\text{ JKM} = \left|\frac{1}{2}\begin{vmatrix} \frac{3}{2} & \frac{1}{2} & 1 \\ \frac{9}{4} & \frac{3}{4} & 1 \\ 2 & 0 & 1 \end{vmatrix}\right| \text{ sq. units}$$

$$= \left|\frac{1}{2}\left\{2\left(\frac{1}{2}-\frac{3}{4}\right)+1\left(\frac{9}{8}-\frac{9}{8}\right)\right\}\right| \text{ sq. units}$$

$$= \left|\frac{1}{2}\left\{-\frac{1}{2}+0\right\}\right| \text{ sq. units}$$

$$= \frac{1}{4} \text{ sq. units}$$

$$\text{Similarly area of } \Delta\text{MKL} = \left|\frac{1}{2}\begin{vmatrix} 2 & 0 & 1 \\ \frac{9}{4} & \frac{3}{4} & 1 \\ \frac{9}{4} & 0 & 1 \end{vmatrix}\right| \text{ sq. units}$$

$$= \left|\frac{1}{2}\left\{\frac{3}{4}\left(2-\frac{9}{4}\right)\right\}\right| \text{ sq. units}$$

$$= \frac{3}{32} \text{ sq. units}$$

$$\therefore \text{ Required area} = \left(\frac{1}{4}+\frac{3}{32}\right) \text{ sq. units}$$

$$= \frac{11}{32} \text{ sq. units}$$

**Hints:**

(i) Draw the graph showing the region bounded by the curves, finding the intersection points.

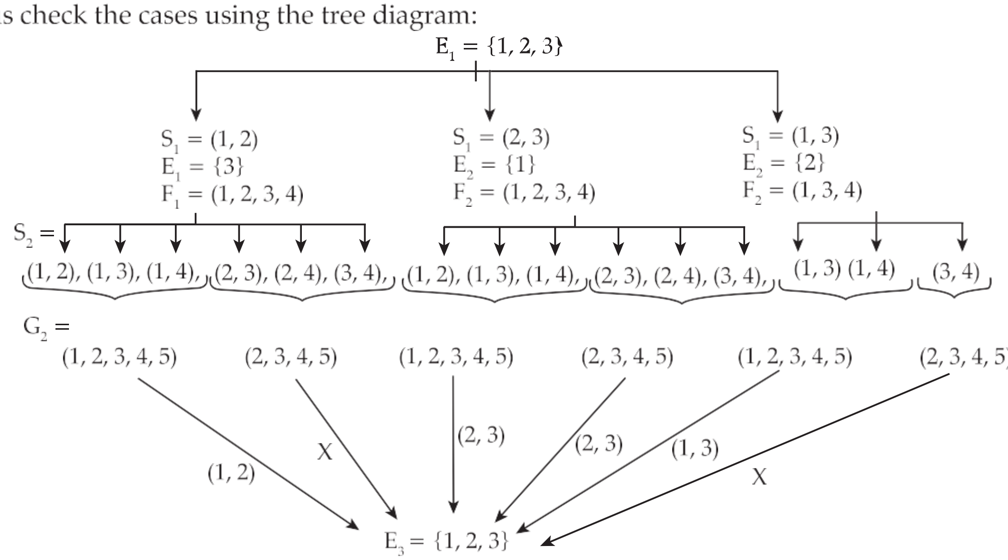
(ii) Use area of triangle choose vertices are $(x_1, y_1)$ $(x_2, y_2)$

And $(x_3, y_3)$ is given by. $\left|\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}\right|$ sq. units

---

**3. Option (A) is correct.**

Let us check the cases using the tree diagram:




$P(E_1 = E_2) = P(S_1 = (1, 2)\ P(S_1 = (2, 3)) + P(S_1 = (1, 3))$

$$= \left(\frac{1}{3}\times\frac{3}{6}\times\frac{1}{{}^5C_2}+0\right)+\left(\frac{1}{3}\times\frac{3}{6}\times\frac{1}{{}^5C_2}+\frac{1}{3}\times\frac{3}{6}\times\frac{1}{{}^4C_2}\right)$$

$$+\left(\frac{1}{3}\times\frac{2}{3}\times\frac{1}{{}^5C_2}+0\right)$$

$$= \frac{1}{12}$$

Required probability

$$= \frac{P\left(S_1=(1,2)\right)\cap\left(E_1=E_3\right)}{P\left(E_1=E_3\right)}$$

$$= \frac{\frac{1}{3} \times \frac{3}{6} \times \frac{1}{^5C_2}}{\frac{1}{12}}$$

$$= \frac{1}{5}$$

**Hints:**

(i) Use the conditional probability

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

(ii) Use the number of ways of selecting $r$ object out of $n$ objects is $^nC_r$

(iii) Use the multiplication rule of probability.

---

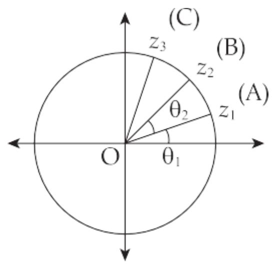
**4. Option (C) is correct.**

Given: $\theta_1 + \theta_2 + \ldots.. + \theta_{10} = 2\pi$

$z_1 = e^{i\theta}, z_k = z_{k-1} e^i \theta_k$

$|z_1| = |z_2| \ldots.. |z_{10}| = 1$

$|z_2 - z_1|$ = Length of line AB




As we know, length of line AB is less than or equal to length of arc AB

$\Rightarrow$ $|z_2 - z_1| \leq$ Length of arc AB

Similarly, $|z_3 - z_2| \leq$ Length of arc BC

$\therefore$ $Q_1 + Q_2 + \ldots\ldots Q_{10} = 2\pi$

$\Rightarrow$ Sum of length of all arcs $= 2\pi$

$\Rightarrow$ $|z_2 - z_1| + |z_3 - z_2| + \ldots.. + |z_1 - z_{10}| \leq 2\pi$

As we know, $|z_k^2 - z_{k-1}^2| = |z_k - z_{k-1}||z_k + z_{k-1}|$

Also, $|z_k + z_{k-1}| \leq |z_k| + |z_{k-1}| \leq 2$

$\Rightarrow$ $|z_k + z_{k-1}||z_k - z_{k-1}| \leq 2|z_k - z_{k-1}|$

$\Rightarrow$ $|z_k^2 - z_{k-1}^2| \leq 2|z_k - z_{k-1}|$

$\Rightarrow$ $|z_2^2 - z_1^2| + |z_3^2 - z_2^2| + \ldots. + |z_1^2 - z_{10}^2| \leq 2 \left[|z_2 - z_1| + |z_3 - z_2| + \ldots. + |z_1 - z_{10}|\right]$

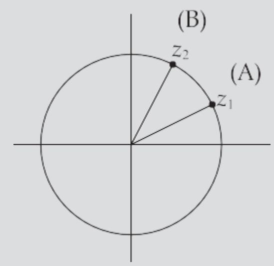
$\Rightarrow$ $|z_2^2 - z_1^2| + |z_3^2 - z_2^2| + \ldots. + |z_1^2 - z_{10}^2| \leq 2(2\pi)$

$\Rightarrow$ $|z_2^2 - z_1^2| + |z_3^2 - z_2^2| + \ldots. + |z_1^2 - z_{10}^2| \leq 4\pi$

**Hints:**

(i) Use $|z_1| = |z_2| \ldots.. |z_{10}| = 1$

(ii) $|z_2 - z_1|$ = length of line A B




(iii) length of arcs $\geq$ length of chord.

(iv) $|z_a^2 - z_b^2| = |z_a - z_b||z_a + z_b|$

**5. Correct answer is [76.25].**

Given: S = {1, 2, 3, ….100}

$p_1$ = probability that the maximum of chosen numbers is at least 81.

$\Rightarrow$ $p_1$ = 1 – probability that the maximum of chosen numbers is at most 80

$\Rightarrow$ $p_1 = 1 - \frac{80 \times 80 \times 80}{100 \times 100 \times 100}$

$\Rightarrow$ $p_1 = 1 - \frac{64}{125}$

$\Rightarrow$ $p_1 = \frac{61}{125}$

$\therefore$ $\frac{625}{4} p_1 = \frac{625}{4} \times \frac{61}{125} = \frac{305}{4} = 76.25$

**Hints:**

(i) Probability that the maximum of chosen numbers is at least 81 = 1 – probability that the maximum of chosen number is at most 80.

**6. Correct answer is [24.50].**

Given: S = {1, 2, 3, ...., 100}

$p_2$ = the probability that the minimum of chosen at most 40.

$\Rightarrow$ $p_2$ = 1 – probability that the minimum of chosen number is at least 41.

$\Rightarrow$ $p_2 = 1 - \frac{60 \times 60 \times 60}{100 \times 100 \times 100}$

$\Rightarrow \quad p_2 = 1 - \frac{27}{125}$

$\Rightarrow \quad p_2 = \frac{98}{125}$

$\Rightarrow \quad \frac{125}{4} p_2 = \frac{125}{4} \times \frac{98}{125} = 24.50$

**Hints:**

(i) The probability that the minimum of chosen number S is at most 40 = 1 – probability that the minimum of chosen numbers is at least 41.

7. **Correct answer is [1].**

Given: The system of liner equation is

$x + 2y + 3z = \alpha$

$4x + 5y + 6z = \beta$

$7x + 8y + 9z = \gamma - 1$

$$D_1 = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$$

$= 1(45 - 48) - 2(36 - 42) + 3(32 - 35)$

$\Rightarrow D_1 = -3 - 2(-6) + 3(-3)$

$\Rightarrow D_1 = 0$

$$D_1 = \begin{vmatrix} \alpha & 2 & 3 \\ \beta & 5 & 6 \\ \gamma - 1 & 8 & 9 \end{vmatrix}$$

$= \alpha(45 - 48) - 2(9\beta - 6\gamma + 6) + 3(8\beta - 5\gamma + 5)$

$\Rightarrow D_1 = -3\alpha - 18\beta + 12\gamma - 12 + 24\beta - 15\gamma + 15$

$\Rightarrow D_1 = -3\alpha + 6\beta - 3\gamma + 3$

$\Rightarrow D_1 = -3(\alpha - 2\beta + \gamma - 1)$

Similarly, $D_2 = \begin{vmatrix} 1 & \alpha & 3 \\ 4 & \beta & 6 \\ 7 & \gamma - 1 & 9 \end{vmatrix}$

$\Rightarrow D_2 = 1(9\beta - 6\gamma + 6) - \alpha(36 - 42) + 3(4\gamma - 4 - 7\beta)$

$\Rightarrow D_2 = 6\alpha - 12\beta + 6\gamma - 6$

And, $D_3 = \begin{vmatrix} 1 & 2 & \alpha \\ 4 & 5 & \beta \\ 7 & 8 & \gamma - 1 \end{vmatrix}$

$= 1(5\gamma - 5 - 8\beta) - 2(4\gamma - 4 - 7\beta) + \alpha(32 - 35)$

$\Rightarrow D_3 = -3\alpha + 6\beta - 3\gamma + 3$

$\Rightarrow D_3 = -3(\alpha - 2\beta + \gamma - 1)$

For system of linear equation to be consistence,

$D_1 = D_2 = D_3 = 0$

Now,

$$M = \begin{vmatrix} \alpha & 2 & \gamma \\ \beta & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$$

$\Rightarrow M = \alpha(1) - 2(\beta) + \gamma(1)$

$\Rightarrow M = \alpha - 2\beta + \gamma$

$\therefore M = 1 \qquad \{\because \alpha - 2\beta + \gamma - 1 = 0\}$

**Hints:**

(i) For a system of linear equations $a_1x + b_1y + c_1z = d$, $a_2x + b_2y + c_2z = d_2$

And $a_3x + b_3y + c_3z = d_3$ where

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}, \Delta_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix},$$

$$\Delta_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}, \Delta_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

To be a consistent system, if $\Delta = 0$ then $\Delta_1 = \Delta_2 = \Delta_3 = 0$

8. **Correct answer is [1.5].**

As from the above question we get,

$\alpha - 2\beta + \gamma - 1 = 0$

$\therefore$ P is the plane containing all those $(\alpha, \beta, \gamma)$ for which the given system of liner equation is consistence.

$\Rightarrow$ The equation of the plane P is

$x - 2y + z - 1 = 0$

Now, distance of the point (0, 1, 0) from the plane $x - 2y + z - 1 = 0$ is

$$\left| \frac{0 - 2(1) + 0 - 1}{\sqrt{(1)^2 + (-2)^2 + (1)^2}} \right|$$

$\Rightarrow \quad \sqrt{D} = \left| \frac{-2 - 1}{\sqrt{6}} \right|$ units

$\Rightarrow \quad \sqrt{D} = \frac{3}{\sqrt{6}}$

$\Rightarrow \quad \left(\sqrt{D}\right)^2 = \left(\frac{3}{\sqrt{6}}\right)^2$

$\Rightarrow \quad D = \frac{9}{6} = \frac{3}{2} = 1.5$

**Hints:**

(i) Use the distance of a point $(x_1, y_1, z_1)$ from the plane $ax + by + cz + d = 0$ is given by $\left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right|$ units

**9. Correct answer is [9].**

Given: Line $L_1: \sqrt{2}x + y - 1 = 0$

Line $L_2: \sqrt{2}x - y + 1 = 0$

Let point P be $(h, k)$

Distance of P $(h, k)$ from $L_1 = \left|\frac{\sqrt{2}h + k - 1}{\sqrt{(\sqrt{2})^2 + (1)^2}}\right|$

$$= \left|\frac{\sqrt{2}h + k - 1}{\sqrt{3}}\right|$$

Similarly, distance of P $(h, k)$ from

$$L_2 = \left|\frac{\sqrt{2}h - k - 1}{\sqrt{3}}\right|$$

Now, $\left|\frac{\sqrt{2}h + k - 1}{\sqrt{3}}\right|\left|\frac{\sqrt{2}h - k + 1}{\sqrt{3}}\right| = \lambda^2$

$\Rightarrow \left|(\sqrt{2}h)^2 - (k-1)^2\right| = 3\lambda^2$

$\Rightarrow$ Locus is $\left|2x^2 - (y-1)^2\right| = 3\lambda^2$

Now, $y = 2x + 1$ cuts C at R and S,

$\Rightarrow \left|2x^2 - (2x + 1 - 1)^2\right| = 3\lambda^2$

$\Rightarrow \left|2x^2 - 4x^2\right| = 3\lambda^2$

$\Rightarrow \left|-2x^2\right| = 3\lambda^2$

$\Rightarrow x = \pm\sqrt{\frac{3}{2}}|\lambda|$

Let R be $(x_1, y_1)$ and S be $(x_2, y_2)$

$\Rightarrow |x_1 - x_2| = \left|\sqrt{\frac{3}{2}}|\lambda| + \sqrt{\frac{3}{2}}|\lambda|\right| = \sqrt{6}|\lambda|$

And $|y_1 - y_2| = |2x_1 + 1 - 2x_2 - 1| = 2|x_1 - x_2|$

$= 2\sqrt{6}|\lambda|$

Now, distance between $R(x_1, y_1)$ and $S(x_2, y_2)$ is $\sqrt{270}$

$\Rightarrow \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{270}$

$\Rightarrow \sqrt{(\sqrt{6}|\lambda|)^2 + (2\sqrt{6}|\lambda|)^2} = \sqrt{270}$

$\Rightarrow 6\lambda^2 + 4(6\lambda^2) = 270$

$\Rightarrow \lambda^2 = \frac{270}{30}$

$\Rightarrow \lambda^2 = 9$

**Hints:**

(i) Use distance of a point $(x_1, y_1)$ from the line $ax + by + c = 0$ is given by $\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$ units

(ii) Use distance formula to find the distance between two points $(x_1, y_1)$ and $(x_2, y_2)$ by $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ units

**10. Correct answer is [77.14].**

Let R be $(x_1, y_1)$ and S be $(x_2, y_2)$

We have $|x_1 - x_2| = \sqrt{6}\lambda$ and

$|y_1 - y_2| = 2\sqrt{6}\lambda$

Slope of line RS $= \frac{y_1 - y_2}{x_1 - x_2} = 2$

Now, R'S' $\perp$ RS

$\Rightarrow$ (slope of RS). (slope of R'S') = –1

$\Rightarrow$ slope of R'S' $= -\frac{1}{2}$

Using mid-point formula, mid-point of RS will be T

$$T = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

As we had, $x_1$ and $x_2 = \pm\sqrt{\frac{3}{2}}|\lambda|$ and

$y_1 = 2x_1 + 1$, $y_2 = 2x_2 + 1$

$\Rightarrow T = \left(\frac{\sqrt{\frac{3}{2}}|\lambda| - \sqrt{\frac{3}{2}}|\lambda|}{2}, \frac{2x_1 + 1 + 2x_2 + 1}{2}\right)$

$\Rightarrow T = (0, x_1 + x_2 + 1)$

$\Rightarrow T = (0, 1)$

So, slope of R'S' $= -\frac{1}{2}$ and R'S' passes through (0, 1)

So, using point slope form of line, equation of R' S' will be,

$$(y - 1) = -\frac{1}{2}(x - 0)$$

$\Rightarrow y - 1 = -\frac{1}{2}x$

Let R' be $(x_3, y_3)$ and S' be $(x_4, y_4)$

Now $y - 1 = -\frac{1}{2}x$ meets curve

$$\left|2x^2 - (y-1)^2\right| = 3\lambda^2 \text{ at } R' \text{ and } S'$$

$$\Rightarrow \left|2x^2 - \left(-\frac{1}{2}x\right)^2\right| = 3\lambda^2$$

$$\Rightarrow \left|\frac{7}{4}x^2\right| = 3\lambda^2$$

$$\Rightarrow x^2 = \frac{12}{7}\lambda^2$$

$$\Rightarrow x = \pm\sqrt{\frac{12}{7}}|\lambda|$$

$$\Rightarrow |x_1 - x_2| = \left|\sqrt{\frac{12}{7}}|\lambda| + \sqrt{\frac{12}{7}}|\lambda|\right| = 2\sqrt{\frac{12}{7}}|\lambda|$$

and

$$|y_1 - y_2| = \left|1 - \frac{1}{2}x_1 - 1 + \frac{1}{2}x_2\right| = \frac{1}{2}|x_2 - x_1| = \sqrt{\frac{12}{7}}|\lambda|$$

Using distance formula,

$$R'S' = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$R'S' = \sqrt{\left(2\sqrt{\frac{12}{7}}|\lambda|\right)^2 + \left(\sqrt{\frac{12}{7}}|\lambda|\right)^2}$$

$$(R'S')^2 = 5 \times \frac{12}{7}(\lambda^2)$$

$$\Rightarrow D = (R'S')^2 = 5 \times \frac{12}{7} \times 9 \quad \{\therefore \lambda^2 = 9\}$$

$$\Rightarrow D = 77.14$$

**Hints:**

(i) Use if slope of two perpendicular lines be $m_1$ and $m_2$ then $m_1 . m_2 = -1$

(ii) Slope of line passing through $(x_1, y_1)$ and $(x_2, y_2)$ is $\frac{y_2 - y_1}{x_2 - x_1}$

(iii) Equation of straight line have slope $m$ and passing through $(x_1, y_1)$ is $(y - y_1) = m(x - x_1)$

(iv) Distance between two points $(x_1, y_1)$ and $(x_2, y_2)$ is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

**11. Options (A), (B) and (D) are correct.**

Given: $|Q| \neq 0$

$$E = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 8 & 13 & 18 \end{vmatrix},\ P = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{vmatrix},\ F = \begin{vmatrix} 1 & 3 & 2 \\ 8 & 18 & 13 \\ 2 & 4 & 3 \end{vmatrix}$$

$$PEP = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix}\begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 8 & 13 & 18 \end{vmatrix}\begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix}$$

$$\Rightarrow PEP = \begin{vmatrix} 1 & 2 & 3 \\ 8 & 13 & 18 \\ 2 & 3 & 4 \end{vmatrix}\begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix}$$

$$\Rightarrow PEP = \begin{vmatrix} 1 & 3 & 2 \\ 8 & 18 & 13 \\ 2 & 4 & 3 \end{vmatrix}$$

$$\Rightarrow PEP = F$$

$$\text{Also, } P^2 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix}\begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\Rightarrow P^2 = I$$

$$\text{Now, } |E| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 8 & 13 & 18 \end{vmatrix}$$

$$= 1(54 - 52) - 2(36 - 32) + 3(26 - 24)$$

$$\Rightarrow |E| = 0$$

$$\text{And } |F| = \begin{vmatrix} 1 & 3 & 2 \\ 8 & 18 & 13 \\ 2 & 4 & 3 \end{vmatrix}$$

$$= 1(54 - 52) - 3(24 - 26) + 2(32 - 36)$$

$$\Rightarrow |F| = 0$$

Now, $|EQ| = |E||Q| = 0$ $\{\therefore |E| = 0\}$

and $\left|PFQ^{-1}\right| = \frac{|P||F|}{|Q|} = 0$ $\{\therefore |F| = 0\}$

let $A = EQ + PFQ^{-1}$

post multiplying the above equation by Q we get,

$$AQ = EQ^2 + PF$$

$$\Rightarrow AQ = EQ^2 + P(PEP) \quad \{\therefore PEP = F\}$$

$$\Rightarrow AQ = EQ^2 + P^2EP$$

$\Rightarrow \quad AQ = EQ^2 + EP \qquad \{\therefore P^2 = I\}$

$\Rightarrow \quad AQ = E(Q^2 + P)$

$\Rightarrow \quad |AQ| = |E||Q^2 + P|$

$\Rightarrow \quad |AQ| = 0 \qquad \{\because |E| = 0\}$

$\Rightarrow \quad |A| = 0 \qquad \{\because |Q| \neq 0\}$

$\Rightarrow \quad |EQ + PFQ^{-1}| = 0$

$\Rightarrow \quad |EQ + PFQ^{-1}| = |EQ| + |PFQ^{-1}|$

Now, as $|E| = 0$, $|F| = 0$

$\Rightarrow \quad |(EF)^3| = 0$, and $|(EF)^2| = 0$

$\therefore \quad 0 > 0$ is not true

$\Rightarrow \quad |(EF)^3| > |EF|^2$ is also not true

Now, $P^2 = I$

$\Rightarrow \quad P^{-1} = P$

$\Rightarrow \quad P^{-1}FP = PFP$

$\Rightarrow \quad P^{-1}FP = P(PEP)P \qquad \{\therefore PEP = F\}$

$\Rightarrow \quad P^{-1}FP = IEI$

$\Rightarrow \quad P^{-1}FP = E$

$\therefore \quad E + P^{-1}FP = E + E = 2E$

And $\quad P^{-1}EP + F = PEP + PEP$

$\{\therefore P^{-1} = P$ and $PEP = F$

$\Rightarrow \quad P^{-1}EP + F = 2PEP$

$\Rightarrow \quad P^{-1}EP + F = 2F$

Now diagonal elements of E are 1, 3, 18 and diagonal elements of F are 1 , 18, 3

$\Rightarrow$ Sum of diagonal elements of E = sum of diagonal elements of F

$\Rightarrow$ Sum of diagonal elements of 2E = sum of diagonal elements of 2F

$\Rightarrow$ Sum of diagonal elements of $E + P^{-1}FP$ = sum of diagonal elements of $P^{-1}EP + F$

**Hints:**

(i) If A is a non singular matrix then $|A| \neq 0$

(ii) Use properties of determinants $|AB| = |A||B|$ and $|AB^{-1}| = \dfrac{|A|}{|B|}$

(iii) Use multiplication of two matrices.

**12. Options (A) and (B) are correct.**

Given: $f(x) = \dfrac{x^2 - 3x - 6}{x^2 + 2x + 4}$

Differentiating both the sides of the above equation we get,

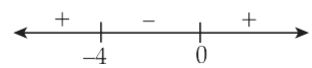
$$f'(x) = \frac{(x^2 + 2x + 4)(2x - 3) - (x^2 - 3x - 6)(2x + 2)}{(x^2 + 2x + 4)^2}$$

$$\Rightarrow \quad f'(x) = \frac{2x^3 - 3x^2 + 4x^2 - 6x + 8x - 12 - 2x^3 - 2x^2 + 6x^2 + 6x + 12x + 12}{(x^2 + 2x + 4)^2}$$

$$\Rightarrow \quad f'(x) = \frac{5x^2 + 20x}{(x^2 + 2x + 4)^2}$$

$$\Rightarrow \quad f'(x) = \frac{5x(x + 4)}{(x^2 + 2x + 4)^2}$$

$\Rightarrow \quad f'(x) = 0$ at $x = 0, -4$




$\therefore \quad f' < 0$ in $(-2, -1)$

$\Rightarrow \quad f(x)$ is decreasing in $(-2, -1)$

$\therefore \quad f'(x) > 0$ in $(1, 2)$

$\Rightarrow \quad f(x)$ is increasing in $(1, 2)$

$\therefore \quad f'(x)$ changes from $+ve$ to $-ve$ at $-4$ so $x = -4$ is the point of maxima and $f'(x)$ changes from $-ve$ to $+ve$ at $x = 0$ so $x = 0$ is the point of minima.

$$f(-4) = \frac{(-4)^2 - 3(-4) - 6}{(-4)^2 + 2(-4) + 4} = \frac{11}{6}$$

And $f(0) = \dfrac{-6}{4} = \dfrac{-3}{2}$

$$\lim_{x \to \pm\infty} f(x) = \lim_{x \to \pm\infty} \frac{x^2 - 3x - 6}{x^2 + 2x + 4} = \lim_{x \to \pm\infty} \frac{1 - \frac{3}{x} - \frac{6}{x^2}}{1 + \frac{2}{x} + \frac{4}{x^2}}$$

$$\Rightarrow \quad \lim_{x \to \pm\infty} f(x) = 1$$

$\Rightarrow$ maximum value of $f(x) = \dfrac{11}{6}$

And minimum value of $f(x) = \dfrac{-3}{2}$

Now, Co-domain of $f(x)$ is R

$\Rightarrow$ Range of $f(x) = \left[\dfrac{-3}{2}, \dfrac{11}{6}\right]$

$\Rightarrow$ Range of $f(x) \neq$ codomain of $f(x)$

$\therefore \quad f$ is into function

**Hints:**

(i) Use quotient rule of differentiation

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2}$$

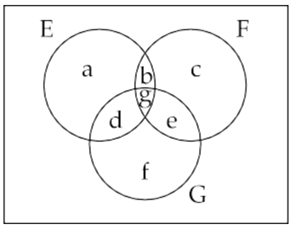
(ii) If $f'(x) < 0$ in an interval then $f(x)$ is decreasing in that interval and if $f'(x) > 0$, then $f(x)$ is increasing.

(iii) A function is onto function if Range = codomain.

**13. Options (A), (B) are C) are correct.**

Given: $P(E) = \frac{1}{8}$, $P(F) = \frac{1}{6}$, $P(G) = \frac{1}{4}$,

$P(E \cap F \cap G) = \frac{1}{10}$




Here, $P(E \cap F \cap G) = g = \frac{1}{10}$

$P(E) = a + b + d + g = \frac{1}{8}$

$\Rightarrow a + b + d = \frac{1}{8} - \frac{1}{10} = \frac{1}{40}$ ...(i)

$P(F) = c + b + e + g = \frac{1}{6}$

$\Rightarrow c + b + e = \frac{1}{6} - \frac{1}{10} = \frac{1}{15}$ ...(ii)

$P(G) = f + d + e + g = \frac{1}{4}$

$\Rightarrow f + d + e = \frac{1}{4} - \frac{1}{10} = \frac{3}{20}$

Now, $P(E \cap F \cap G^c) = b$

From eq. (i) $b \le \frac{1}{40}$

$\Rightarrow P(E \cap F \cap G^c) \le \frac{1}{40}$

Similarly, $P(E^c \cap F \cap G) = e$

From eq. (ii) $e \le \frac{1}{15}$

$\Rightarrow P(E \cap F \cap G) \le \frac{1}{15}$

Also, $P(E \cup F \cup G) \le P(E) + P(F) + P(G)$

$\Rightarrow P(E \cup F \cup G) \le \frac{1}{8} + \frac{1}{6} + \frac{1}{4}$

$\Rightarrow P(E \cup F \cup G) \le \frac{13}{24}$

By De Morgan's Law, $P(E^c \cap F^c \cap G^c)$

$= P(E \cup F \cup G)^c$

$\Rightarrow P(E^c \cap F^c \cap G^c) \ge 1 - \frac{13}{24}$

$\Rightarrow P(E^c \cap F^c \cap G^c) \ge \frac{11}{24}$

**Hints:**

(i) Use venn diagram for three variables and solve it.

(ii) Use De Morgan's Law $P(A \cup B \cup C) = P(A^c \cap B^c \cap C^c)$

(iii) Use complement rule of probability, $1 - P(A) = P(A^c)$

**14. Options (A), (B) and C) are correct.**

Given: $I - EF$ is invertible matrix

$G = (I - EF)^{-1}$

$\Rightarrow G^{-1} = I - EF$ ...(i)

Post multiplying by G, we get,

$G^{-1}G = IG - EFG$

$\Rightarrow I = G - EFG$ ...(ii)

Pre multiplying equation (i) by G we get,

$GG^{-1} = GI - GEF$

$\Rightarrow I = G - GEF$ ...(iii)

From equation (ii) and (iii), we get

$EFG = GEF$

Now, $(I - FE)(I + FGE)$

$= I + FGE - FE - FEFGE$

$= I + FGE - FE - F(G - I)E$

$= I + FGE - FE - FGE + FE$

$= I$

$\Rightarrow (I - FE)(I + FGE) = I$

And $(I - FE)(I - FGE)$

$= I - FE - FGE + FE\,FGE$

$= I - FE - FGE + F(G - I)E$

$= I - FE - FGE + FGE - FE$

$= I - 2FE$

$\Rightarrow (I - FE)(I - FGE) \ne I$

Now, $(I - FE)(FGE) = FGE - FEFGE$

$\Rightarrow (I - FE)(FGE) = FGE - F(G - I)E$

$\Rightarrow (I - FE)(FGE) = FGE - FGE + FE$

$\Rightarrow (I - FE)(FGE) = FE$

$\Rightarrow |I - FE||FGE| = |FE|$

**Hints:**

(i) Similarly using identities of multiplication of matrices.

(ii) Use properties of inverse of matrix $AA^{-1} = I = A^{-1}A$

**15. Options (A) and (B) are correct.**

Given: $S_n(x) = \sum_{k=1}^{n} \cot^{-1}\left(\frac{1+k(k+1)x^2}{x}\right) \forall x \in R$

$\Rightarrow S_n(x) = \sum_{k=1}^{n} \tan^{-1}\left(\frac{x}{1+k(k+1)x^2}\right)$

$\left\{\because \cot^{-1} x = \tan^{-1}\frac{1}{x}\right\}$

$\Rightarrow S_n(x) = \sum_{k=1}^{n} \tan^{-1}\left(\frac{kx+x-kx}{1+(kx+x)kx}\right)$

$\Rightarrow S_n(x) = \sum_{k=1}^{n}\left[\tan^{-1}(kx+x) - \tan^{-1}(kx)\right]$

$\left\{\because \tan^{-1} A - \tan^{-1} B = \tan^{-1}\left(\frac{A-B}{1+AB}\right)\right\}$

$\Rightarrow S_n(x) = \sum_{k=1}^{n}\left\{\tan^{-1}((k+1)x) - \tan^{-1}(kx)\right\}$

$\Rightarrow S_n(x) = \tan^{-1} 2x - \tan^{-1} x + \tan^{-1} 3x - \tan^{-1} 2x + \ldots + \tan^{-1}(n+1)x - \tan^{-1} nx$

$\Rightarrow S_n(x) = \tan^{-1}(nx + x) - \tan^{-1} x$

$\Rightarrow S_n(x) = \tan^{-1}\left(\frac{nx}{1+(n+1)x^2}\right)$

$\Rightarrow S_{10}(x) = \tan^{-1}\left(\frac{10x}{1+11x^2}\right)$

$\Rightarrow S_{10}(x) = \frac{\pi}{2} - \cot^{-1}\left(\frac{10x}{1+11x^2}\right)$

$\left\{\because \tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}\right\}$

$\Rightarrow S_{10}(x) = \frac{\pi}{2} - \tan^{-1}\left(\frac{1+11x^2}{10x}\right) \forall x > 0$

$\Rightarrow$ Now, $\lim_{n\to\infty} \cot(S_n(x)) = \lim_{n\to\infty} \cot\left[\tan^{-1}\left(\frac{nx}{1+(n+1)x^2}\right)\right]$

$\Rightarrow \lim_{n\to\infty} \cot(S_n(x)) = \lim_{n\to\infty} \cot\left[\cot^{-1}\left(\frac{1+(n+1)x^2}{nx}\right)\right]$

$\Rightarrow \lim_{n\to\infty} \cot(S_n(x)) = \lim_{n\to\infty}\left(\frac{1+(n+1)x^2}{nx}\right)$

$\{\because \cot(\cot^{-1} x) = x\}$

$\Rightarrow \lim_{n\to\infty} \cot(S_n(x)) = \lim_{n\to\infty}\left(\frac{\frac{1}{n}+\left(1+\frac{1}{n}\right)x^2}{x}\right)$

$\Rightarrow \lim_{n\to\infty} \cot(S_n(x)) = \frac{x^2}{x} = x \forall x > 0$

$\Rightarrow$ Now, $S_3(x) = \tan^{-1}\left(\frac{3x}{1+4x^2}\right)$

$\Rightarrow$ So, for equation $S_3(x) = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\frac{3x}{1+4x^2}\right) = \frac{\pi}{4}$

$\Rightarrow \frac{3x}{1+4n^2} = \tan\frac{\pi}{4}$

$\Rightarrow \frac{3x}{1+4x^2} = 1$

$\Rightarrow 4x^2 - 3x + 1 = 0$

$\Rightarrow x = \frac{3x \pm \sqrt{9-16}}{8}$

{Using Shri dharacharya's rule}

$\Rightarrow x \notin R$

So, equation $S_3(x) = \frac{\pi}{4}$ has no root in $(0, \infty)$

Now, $\tan(S_n(x)) = \tan\left(\tan^{-1}\left(\frac{nx}{1+(n+1)x^2}\right)\right)$

$\Rightarrow \tan(S_n(x)) = \frac{nx}{1+(n+1)x^2}$

$\forall \cap \geq 1; x > 0; \cap \in I^+$

$\Rightarrow$ Now for $\tan(S_n(x)) \leq \frac{1}{2}$

$\Rightarrow \frac{nx}{1+(n+1)x^2} \leq \frac{1}{2}$

$\Rightarrow 2nx \leq 1 + (n+1)x^2$

$\Rightarrow (n+1)x^2 - 2nx + 1 \geq 0$

Discriminant of $(n+1)x^2 - 2nx + 1 = 0$ is

$D = (-2n)^2 - 4(n+1)(1)$

$D = 4n^2 - 4n - 4$

$D = 4n^2 - 4(n+1)$

For $n = 1$, $D = -4 \quad \Rightarrow D < 0$

$\Rightarrow (n+1)x^2 - 2nx + 1 > 0$

For $n \geq 2$, $D > 0$

For some $x > 0$, $(n+1)x^2 - 2nx + 1 < 0$

$\therefore$ tan ($S_n$ ($x$)) $\leq \frac{1}{2}$ $\forall$ $n \geq 1$ and $x > 0$ is not true.

**Hints:**

(i) use identities of inverse trigonometric functions

$\cot^{-1} x = \tan^{-1} \frac{1}{x}$, $\tan^{-1} A - \tan^{-1} B = \tan^{-1} \left(\frac{A-B}{1+AB}\right)$

$\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$, $\cot(\cot^{-1} x) = x$

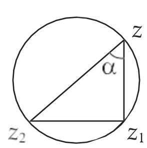
(ii) find roots of quadratic equation using Sridharacharya rule i.e. roots of $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2 - 4ax}}{2a}$

(iii) use nature of roots of quadrate equation using discrimina for $ax^2 + bx + c = 0$, if D $= b^2 - 4ax < 0$, then $ax^2 + bx + c > 0$ and if D > 0 then $x$ has real values so for some $x$ $ax^2 + bx + c < 0$.

**16. Options (B) and D) are correct.**

Given: $\arg\left(\frac{z-\alpha}{z+\beta}\right) = \frac{\pi}{4}$

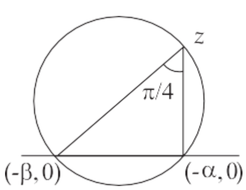
As we know, if $\arg\left(\frac{z+z_1}{z+z_2}\right) = \alpha$ where $\alpha \in (0, \pi)$




$\Rightarrow$ $z$ would lie on an arc of segment of a circle on $z_1 z_2$ containing angle $\alpha$.

So, $\arg\left(\frac{z+\alpha}{z+\beta}\right) = \frac{\pi}{4}$

$\Rightarrow$ $z$ lie on an arc of segment of a circle with $(-\alpha, 0)$ and $(-\beta, 0)$ and subtend an angle $\frac{\pi}{4}$ on $z$




Also, $z$ lie on circle $x^2 + y^2 + 5x - 3y + 4 = 0$

$\Rightarrow$ $(-\alpha, 0)$ and $(-\beta, 0)$ also lie on the circle

So, $y = 0$, $x^2 + 5x + 4 = 0$

$\Rightarrow$ $(x + 1)(x + 4) = 0$

$\Rightarrow$ $x = -1, -4$

$\Rightarrow$ $\alpha = 1$ and $\beta = 4$

$\therefore$ $\alpha\beta = 4$ and $\beta = 4$

**Hints:**

(i) If $\arg\left(\frac{z-z_1}{z-z_2}\right) = \alpha$ where $\alpha$ whose $\alpha \in (0, \pi)$

then $z$ would lie on an arc of segment of a circle on $z_1 z_2$ containing angle $\alpha$

**17. Correct answer is [4].**

Given: $3x^2 - 4|x^2 - 1| + x - 1 = 0$

For $x \in [-1, 1]$ $\quad |x^2 - 1| = -(x^2 - 1)$

$\therefore$ $3x^2 + 4(x^2 - 1) + x - 1 = 0$

$\Rightarrow$ $7x^2 + x - 5 = 0$

$\Rightarrow$ $x = \frac{(-1) \pm \sqrt{1 + 4(5)(7)}}{2(7)}$

(Using Sridharacharaya's rule)

$\Rightarrow$ $x = \frac{-1 \pm \sqrt{141}}{14}$

$\Rightarrow$ Both values of $x \in [-1, 1]$

Now, for $x \in (-\infty, -1) \cup (1, \infty)$

$|x^2 - 1| = -(x^2 - 1)$

$\therefore$ $3x^2 - 4(x^2 - 1) + x - 1 = 0$

$\Rightarrow$ $x^2 - x - 3 = 0$

$\Rightarrow$ $x = \frac{-(-1) \pm \sqrt{1 - 4(-3)}}{2}$

$\Rightarrow$ $x = \frac{1 \pm \sqrt{13}}{2}$

$\Rightarrow$ Both values $x \in (-\infty, -1) \cup (1, \infty)$

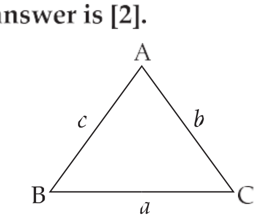
$\therefore$ given equation has 4 real roots.

**Hints:**

(i) Use definition of modulus function.

(ii) Use Sridharacharya's rule for $ax^2 + bx + c = 0$ roots are given by $x = \frac{-b \pm \sqrt{b^2 - 4ax}}{2a}$

**18. Correct answer is [2].**




Given: $\Delta$ABC such that

AB = $\sqrt{23}$, BC = 3, CA = 4

$\Rightarrow$ $c = \sqrt{23}$, $a = 3, b = 4$

$$\cot A = \frac{\cos A}{\sin A} = \frac{b^2 + c^2 - a^2}{2bc \sin A}$$

$\therefore$ Area of triangle $\Delta ABC = \Delta = \frac{1}{2} bc \sin A$

$$\Rightarrow \cot A = \frac{b^2 + c^2 - a^2}{4\Delta}$$

Similarly, $\cot B = \frac{a^2 + c^2 - b^2}{4\Delta}$ and

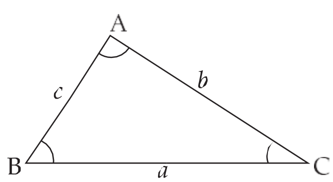
$$\cot C = \frac{a^2 + b^2 - c^2}{4\Delta}$$

$$\therefore \frac{\cot A + \cot C}{\cot B} = \frac{\frac{b^2 + c^2 - a^2}{4\Delta} + \frac{a^2 + b^2 - c^2}{4\Delta}}{\frac{a^2 + c^2 - b^2}{4\Delta}}$$

$$\Rightarrow \frac{\cot A + \cot C}{\cot B} = \frac{2b^2}{a^2 + c^2 - b^2}$$

$$= \frac{2(4)^2}{(3)^2 + (\sqrt{23})^2 - (4)^2}$$

$$\Rightarrow \frac{\cot A + \cot C}{\cot B} = 2$$




**Hints:**

(i) Use cosine formula $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

(ii) Use area of triangle $= \frac{1}{2} bc \sin A = \frac{1}{2} ab \sin c = \frac{1}{2} ac \sin B$

**19. Correct answer is [7].**

Given: $|\vec{u}| = 1, |\vec{v}| = 1$

$\vec{u}$ is not perpendicular to $\vec{v}$

$\vec{u}.\vec{v} \neq 0$

And $\vec{u}.\vec{w} = 1$ $\vec{v}.\vec{w} = 1$ $\vec{w}.\vec{w} = 4$

$\Rightarrow$ $\vec{w}.\vec{w} = |\vec{w}|^2 = 4$

$\Rightarrow$ $|\vec{w}| = 2$

Volume of parallelepiped with $\vec{u}, \vec{v}$ and $\vec{w}$ as its sides,

$$= [\vec{u}\,\vec{v}\,\vec{w}] = \sqrt{2}$$

Now, $[\vec{u}\,\vec{v}\,\vec{w}]^2 = \begin{vmatrix} \vec{u}.\vec{u} & \vec{u}.\vec{v} & \vec{u}.\vec{w} \\ \vec{v}.\vec{u} & \vec{v}.\vec{v} & \vec{v}.\vec{w} \\ \vec{w}.\vec{u} & \vec{w}.\vec{v} & \vec{w}.\vec{w} \end{vmatrix} = 2$

$$\Rightarrow \begin{vmatrix} 1 & \vec{u}.\vec{v} & 1 \\ \vec{v}.\vec{u} & 1 & 1 \\ 1 & 1 & 4 \end{vmatrix} = 2$$

$$\Rightarrow 1(4-1) - \vec{u}.\vec{v}(4\vec{u}.\vec{v} - 1) + 1(\vec{u}.\vec{v} - 1) = 2$$

$$\Rightarrow 3 - 4(\vec{u}\,\vec{v})^2 + \vec{u}\,\vec{v} + \vec{u}\,\vec{v} - 1 = 2$$

$$\Rightarrow -4(\vec{u}\,\vec{v})^2 + 2\vec{u}\,\vec{v} + 2 = 2$$

$$\Rightarrow -4(\vec{u}\,\vec{v})^2 + 2\vec{u}\,\vec{v} = 0$$

$$\Rightarrow 2\vec{u}\,\vec{v}(-2\vec{u}\,\vec{v} + 1) = 0$$

$$\Rightarrow \vec{u} \cdot \vec{v} = \frac{1}{2}, \vec{u} \cdot \vec{v} \neq 0$$

Now, $|3\vec{u} + 5\vec{v}| = \sqrt{9 + 25 + 30\left(\frac{1}{2}\right)}$

$$\Rightarrow |3\vec{u} + 5\vec{v}| = \sqrt{49}$$

$$\Rightarrow |3\vec{u} + 5\vec{v}| = 7$$

**Hints:**

(i) If two vectors $\vec{a}$ and $\vec{b}$ are not perpendicular then $\vec{a}\,\vec{b} \neq 0$

(ii) Volume of parallelepiped with $\vec{a}, \vec{b}, \vec{c}$ as its sides is $[\vec{a}\,\vec{b}\,\vec{c}]$

(iii) $[\vec{a}\,\vec{b}\,\vec{c}]^2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{vmatrix}$